61.(3) Let $I_n = \int_0^1 e^x (x-1)^n \, dx$

Put $x - 1 = t \quad \Rightarrow \quad dx = dt$

$$\therefore \quad I_n = \int_{-1}^{0} e^{t+1} \cdot t^n dt = e\int_{-1}^{0} t^n e^t dt$$

$$= e\left([t^n e^t]_{-1}^{0} - n\int_{-1}^{0} t^{n-1} e^t dt \right) = e\left(0 - (-1)^n e^{-1} - n\int_{-1}^{0} t^{n-1} e^t dt \right)$$

$$= (-1)^{n+1} - ne\int_{-1}^{0} t^{n-1} e^t dt \quad \Rightarrow \quad I_n = (-1)^{n+1} - nI_{n-1} \qquad \text{........ (i)}$$

For $\quad n = 1,\ I_1 = \int_0^1 e^x (x-1) dx = [e^x (x-1)]_0^1 - \int_0^1 e^x dx$

$$= e^1 (1-1) - e^0 (0-1) - [e^x]_0^1 = 1 - (e-1) = 2 - e$$

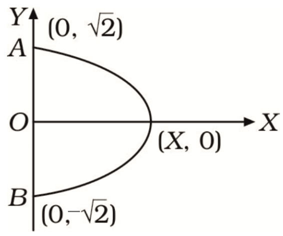
Therefore, from Eq. (i), we get

$$I_2 = (-1)^{2+1} - 2I_1 = -1 - 2(2-e) = 2e - 5$$

and $\quad I_3 = (-1)^{3+1} - 3I_2 = 1 - 3(2e-5) = 16 - 6e$

Hence, $n = 3$ is the answer.

62. Since, f is continuous function and $\int_0^x f(t)dt \to \infty$, as $|x| \to \infty$



To show that every line $y = mx$ intersects the curves

$$y^2 + \int_0^x f(t)dt = 2$$

At $x = 0, y = \pm\sqrt{2}$

Hence, $(0, \sqrt{2}), (0, -\sqrt{2})$ are the point of intersection of the curve with the Y-axis.

As, $x \to \infty$, $\int_0^x f(t)dt \to \infty$ for a particular x (say x_n), then $\int_0^x f(t)dt = 2$ and for this value of $x, y = 0$

The curve is symmetrical about, X-axis.

Thus, we have that there must be some x, such that $f(x_n) = 2$.

Thus, $y = mx$ intersects this closed curve for all values of m.

63. Given, $f(x) = \int_1^x [2(t-1)(t-2)^3 + (t-1)^2 (t-2)^2] dt$

$$\therefore \quad f'(x) = [2(x-1)(x-2)^3 + 3(x-1)^2 (x-2)^2] \cdot 1 - 0$$

$$= (x-1)(x-2)^2 [2(x-2) + 3(x-1)]$$

$$= (x-1)(x-2)^2 (5x-7)$$

$+ \quad - \quad +$

$\therefore \quad f(x)$ attains maximum at $x = 1$ and $f(x)$ attains minimum at $x = \frac{7}{5}$

64.(BD) Here, $\lim_{n\to\infty}\dfrac{1^a+2^a+.....+n^a}{(n+1)^{a-1}\{(na+1)+(na+2)+.....+(na+n)\}}=\dfrac{1}{60}$

$\Rightarrow \quad \lim_{n\to\infty}\dfrac{\sum_{r=1}^{n} r^a}{(n+1)^{a-1}\cdot\left[n^2a+\dfrac{n(n+1)}{2}\right]}=\dfrac{1}{60} \quad \Rightarrow \quad \lim_{n\to\infty}\dfrac{2\sum_{r=1}^{n}\left(\dfrac{r}{n}\right)^a}{\left(1+\dfrac{1}{n}\right)^{a-1}\cdot(2na+n+1)}=\dfrac{1}{60}$

$\Rightarrow \quad \lim_{n\to\infty}\dfrac{1}{n}\left(2\sum_{r=1}^{n}\left(\dfrac{r}{n}\right)^a\right)\cdot\lim_{n\to\infty}\dfrac{1}{\left(1+\dfrac{1}{n}\right)^{a-1}\cdot\left(2a+1+\dfrac{1}{n}\right)}=\dfrac{1}{60}$

$\Rightarrow \quad 2\int_0^1 (x^a)\,dx\cdot\dfrac{1}{1\cdot(2a+1)}=\dfrac{1}{60} \quad \Rightarrow \quad \dfrac{2\cdot[x^{a+1}]_0^1}{(2a+1)\cdot(a+1)}=\dfrac{1}{60}$

$\therefore \quad \dfrac{2}{(2a+1)(a+1)}=\dfrac{1}{60} \quad \Rightarrow \quad (2a+1)(a+1)=120 \quad \Rightarrow \quad 2a^2+3a+1-120=0$

$\Rightarrow \quad 2a^2+3a-119=0 \quad \Rightarrow \quad (2a+17)(a-7)=0 \quad \Rightarrow \quad a=7,\dfrac{-17}{2}$

65.(AD) Given, $S_n=\sum_{k=0}^{n}\dfrac{n}{n^2+kn+k^2} \quad =\sum_{k=0}^{n}\dfrac{1}{n}\cdot\left(\dfrac{1}{1+\dfrac{k}{n}+\dfrac{k^2}{n^2}}\right)<\lim_{n\to\infty}\sum_{k=0}^{n}\dfrac{1}{n}\left(\dfrac{1}{1+\dfrac{k}{n}+\left(\dfrac{k}{n}\right)^2}\right)$

$=\int_0^1\dfrac{1}{1+x+x^2}dx=\left[\dfrac{2}{\sqrt{3}}\tan^{-1}\left(\dfrac{2}{\sqrt{3}}\left(x+\dfrac{1}{2}\right)\right)\right]_0^1=\dfrac{2}{\sqrt{3}}\cdot\left(\dfrac{\pi}{3}-\dfrac{\pi}{6}\right)=\dfrac{\pi}{3\sqrt{3}}$

i.e. $S_n<\dfrac{\pi}{3\sqrt{3}}$ Similarly, $T_n>\dfrac{\pi}{3\sqrt{3}}$

66. $\lim_{n\to\infty}\left(\dfrac{1}{n+1}+\dfrac{1}{n+2}+.....+\dfrac{1}{6n}\right)=\sum_{r=1}^{5n}\dfrac{1}{n+r}=\lim_{n\to\infty}\dfrac{1}{n}\sum_{r=1}^{5n}\left(\dfrac{1}{1+\dfrac{r}{n}}\right)$

$=\int_0^5\dfrac{dx}{1+x}=[\log(1+x)]_0^5=\log 6-\log 1=\log 6$

67.(D) Let equation of tangent to parabola be $y=mx+\dfrac{2}{m}$

It also touches the circle $x^2+y^2=2$.

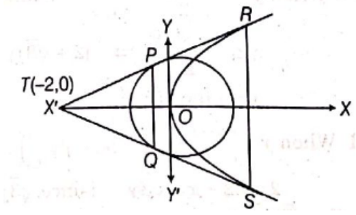
$\therefore \quad \left|\dfrac{2}{m\sqrt{1+m^2}}\right|=\sqrt{2}$

$\Rightarrow \quad m^4+m^2=2 \quad \Rightarrow \quad m^4+m^2-2=0$

$\Rightarrow \quad (m^2-1)(m^2+2)=0 \quad \Rightarrow \quad m=\pm1, m^2=-2$ [rejected $m^2=-2$]

So, tangents are $y=x+2, y=-x-2$.

They, intersect at $(-2, 0)$.



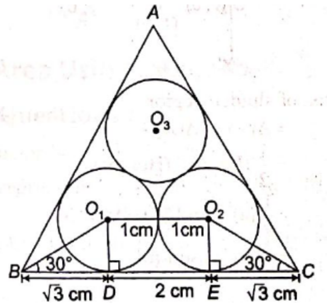
Equation of chord PQ is $-2x = 2 \Rightarrow x = -1$

Equation of chord RS is $O = 4(x-2) \Rightarrow x = 2$

$\therefore$ Coordinates of P, Q, R, S are $P(-1,1), Q(-1,-1), R(2,4), S(2,-4)$

$\therefore$ Area of quadrilateral $= \dfrac{(2+8)\times 3}{2} = 15$ sq units

68.(A) Since, tangents drawn from external points to the circle subtends equal angle at the centre.



$\therefore \quad \angle O_1BD = 30°$

In ΔO_1BD, $\tan 30° = \dfrac{O_1D}{BD} \Rightarrow BD = \sqrt{3}\ cm$

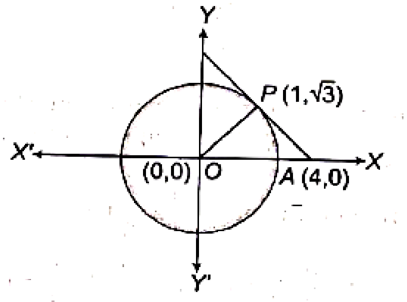
Also, $DE = O_1O_2 = 2cm$ and EC = $\sqrt{3}\ cm$

Now, $BC = BD + DE + EC = 2 + 2\sqrt{3}$

$\Rightarrow$ Area of $\Delta ABC = \dfrac{\sqrt{3}}{4}(BC)^2 = \dfrac{\sqrt{3}}{4}\cdot 4\left(1+\sqrt{3}\right)^2 = \left(6+4\sqrt{3}\right)\ sq\ cm$.

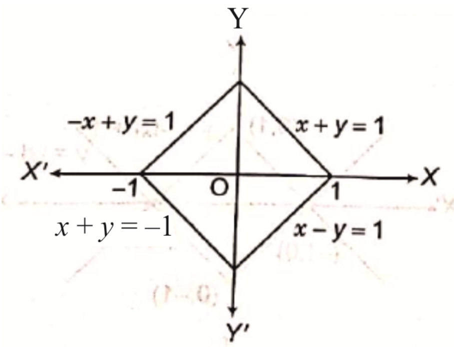
69. $2\sqrt{3}$ Equation of tangent at the point $\left(1, \sqrt{3}\right)$ to the curve $x^2 + y^2 = 4$ is $x + \sqrt{3}y = 4$

Whose X-axis intercept (4, 0).



Thus, area of Δ formed by (0, 0) $\left(1, \sqrt{3}\right)$ and (4, 0) $= \dfrac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 1 & \sqrt{3} & 1 \\ 4 & 0 & 1 \end{vmatrix} = \dfrac{1}{2}\left|\left(0 - 4\sqrt{3}\right)\right| = 2\sqrt{3}$ square units.

70.(2) The area formed by $|x| + |y| = 1$ is square shown as below:



$\therefore$ Area of square $= \left(\sqrt{2}\right)^2 = 2$ square units.